

# RAPID PROTOTYPING WITH SLOPING SURFACES

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## ABSTRACT

*TruSurf is a new system for building solid objects from layers with sloping surfaces that closely match the designed surface shape. The advantages of using sloping surfaces over stepped edges are improved surface finish, and decreased build time through the use of thicker layers. TruSurf uses B-spline surfaces to describe the part, and calculate the sloped path of the layer cutting medium. The operation of the system is described in detail, and results from the production of some test parts are presented. Finally, some ways for improving accuracy are discussed, including using principal directions of minimum surface curvature, and using a curved cutting medium to produce layers.*

## INTRODUCTION

Commercial layer building Rapid Prototyping (RP) systems, such as stereolithography, selective laser sintering, fused deposition modeling, and laminated object manufacturing, use cross sections with edges square to the layer plane (Jacobs 1992). This creates a stepped effect on the surface of the prototype. To improve the finish of a part, this study has been undertaken to build layers with surfaces that closely match the surface slope of the original CAD model. The difficulties with using sloped surfaces stem from obtaining the data required to create the part. It is much easier to obtain the outline of a cross section than to obtain its outer surface information. Using sloped surfaces also means that a four or five axis controller is needed on the layer cutter, and this adds to the cost of the system.

With respect to the production of an actual prototype, the use of sloped surfaces on layers has two main advantages. Firstly it can improve the surface of the model and thus reduce the need for finishing. There has been some recent work on developing laminated metal tooling for use in injection moulding (Dickens 1996). The layers are cut with edges square to the layer plane. Typically the laminates are cut from 1 mm or 0.5 mm thick sheet steel with a laser, plasma, or waterjet cutter. Due to the thickness of the steel, the stair case effect on the tool is a problem. In this situation the use of sloping surfaces on the layers is one possible solution. The rapid prototyping group at Nottingham University have developed another interesting solution (Soar et al 1996). They use copper coated Stereolithography models as EDM electrodes to remove the steps from their laminated tools. This finishing process has the potential to produce a significantly improved surface, but it is also more costly and time consuming.

The second advantage of producing prototypes with sloping surfaces is that, when parts with no intricate detail are being built, it allows the use of thicker layers, thus decreasing the build time. This is particularly advantageous for large parts. The major problem with building parts larger than one meter cubed is the build time. For example to build an object such as a full size sailing skiff would take as much as 15 days using current rapid prototyping systems with typical build rates of 15 mm per hour. Using sloping surfaces and much thicker layers could bring the build time down to reasonable levels, while still producing an acceptable surface.

The system presented here was initially proposed by Hope et al (1995) and Hope et al (1996). It was mainly concerned with building objects larger than one meter cubed, but can be used for smaller parts. The name "True Surface System" or TruSurf was chosen for two reasons. Firstly, it takes advantage of direct slicing of B-spline surfaces from a CAD model instead of using an intermediate faceted STL (Jacobs 1992) file format. Secondly, a truer surface is created on the part by using sloped surfaces.

## DIRECT SLICING OF CAD MODELS

Rapid prototyping machines obtain the 3D geometry, needed to build parts, from CAD models. The number of different CAD systems and file formats in use today means that it is unreasonable to expect RP machines to be able to process all of them. Thus a universal form of input from CAD to RP is required. Most RP systems use a faceted file

format as a means to obtain the model geometry, and the de facto standard is STL. The faceted model is created by mapping a triangular mesh over a surface or solid model. This produces an approximation to the original CAD model and can affect the quality of the surface of the final product.

Manufacturing of objects with RP processes can only be as accurate as the data transmitted to the prototyping apparatus. Thus the accuracy of this data is of major importance to the quality of the part produced. This has led to work on direct slicing of CAD models, to retain as much as possible of the designers intended geometry. At Cranfield University (Jamieson & Hacker 1995), Parasolid models within the Unigraphics CAD package have been directly sliced into Hewlett-Packard Graphics Language (HPGL), and Common Layer Interface (CLI) files. Rajagopalan et al (1995), at Clemson University, have directly sliced the Non-Uniform Rational B-spline Surfaces (NURBS) in the I-DEAS CAD system using the I-DEAS language.

The main advantages of direct slicing of models within CAD software, as opposed to slicing of an intermediate faceted model are, greater model accuracy, and reduced RP machine pre-processing time. The potential disadvantages are that the ability to reorientate the model is lost, and supports cannot be easily added to the sliced model.

TruSurf uses IGES files (U.S. Standards 1988) exported from a CAD system to obtain its 3D geometry. IGES stores its surfaces as NURBS, and TruSurf uses these surface descriptions directly to trace contours and perform slope calculations. NURBS use the same geometric definitions for surfaces as the original CAD model, thus accuracy is maintained. Since the slicing is not performed within the CAD software the model can still be reoriented, and if needed, supports can be added before slicing.

## **TRUE SURFACE SYSTEM**

The TruSurf system was implemented in C++ as a stand alone program and operates independently from a CAD system. It can handle multiple surfaces from the one IGES file to define a part. First TruSurf reads the IGES file and stores the B-spline surfaces in memory. It checks the surfaces to find their maximum and minimum dimensions in each of the x, y, and z directions. This information is displayed to the user. At this stage the user is required to choose the orientation of the slicing plane with respect to the CAD model, and what layer thickness to use. TruSurf slices the model by tracing surface contours, and computing the cutting direction at a number of points, specified by the user, around the contour. These points and corresponding cutting vectors are used to create Numerical Control (NC) code for the machine that will be used to cut the layers.

At present this project has access to a five-axis waterjet cutter at the Queensland Manufacturing Institute (QMI). The five axes are X, Y, Z, A, and B, where X, Y, Z are the 3D Cartesian co-ordinates of a point from a designated origin, and A, B are head rotation angles. When cutting the layers, the waterjet moves linearly between points. Thus the accuracy of the surface produced is dependent on the number of points used, and their spacing on the perimeter of each contour. The number of points used by TruSurf is selected by the user. Obviously, using more points leads to a more accurate surface. The number of points used has little effect on the cutting time, as it is dependent primarily on the traverse speed of the waterjet. However using more points increases the size of the NC code files produced. This can be a problem, as QMI's waterjet control system is limited in the size of file it can handle. Practically, this limited the present study to about one thousand four hundred points per file.

## **SLICING B-SPLINE SURFACES**

Within IGES files, solid and surface models may be represented as rational B-spline surfaces. TruSurf uses these surfaces because they enable calculation of surface slope and curvature. They are also closer to the representation of the model surface within the CAD system than the faceted approximations used by most RP systems. A rational B-spline surface (see for example Rogers & Adams 1990) is expressed parametrically in the form,

$$P(s,t) = \frac{\sum_{i=1}^{n+1} \sum_{j=1}^{m+1} W_{i,j} P_{i,j} b_{i,k}(s) b_{j,l}(t)}{\sum_{i=1}^{n+1} \sum_{j=1}^{m+1} W_{i,j} b_{i,k}(s) b_{j,l}(t)} \tag{1}$$

with parameters s and t ranging from zero to one. The  $W_{i,j}$  terms are weights, the  $P_{i,j}$  terms are 3D net points of the defining polygon, and  $b_{i,k}$  and  $b_{j,l}$  are B-spline basis functions of order k and l respectively. The B-spline basis functions are defined by the Cox-deBoor recursion formulas.

$$b_{i,k}(s) = \begin{cases} 1 & \text{if } x_j \leq t \leq x_{j+1} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and

$$b_{i,k}(s) = \frac{(s - x_j) b_{i,k-1}(s)}{x_{i+k-1} - x_j} + \frac{(x_{i+k} - s) b_{i+1,k-1}(s)}{x_{i+k} - x_{i+1}} \tag{3}$$

The values of  $x_i$  are elements of a knot vector satisfying the relation  $x_j \leq x_{j+1}$ , and the convention  $0/0 = 0$  is adopted.

The IGES file contains the weights, net points, knot vectors, degree of both basis functions, and upper index of both sums. With this information a 3D point on the surface is specified by its parameters s and t. To slice the B-spline surfaces, the model is first reorientated so that the normal of the cutting planes, specified by the user, becomes the z axis of the CAD model. TruSurf traces contours of constant z height, at spacings equivalent to layer thickness. Each contour start point is found by setting parameter s to zero, and finding the value of t that gives a z coordinate equal to the current layer height. An iterative function solving method with adaptive step size is used to find t. Once s and t are known the x and y coordinates can be found. The contour is then traced by increasing parameter s by the inverse of the number of points used, and finding the new t value to satisfy the contour height.

Once the contour points are known, extra information is required to orientate the cutting vectors and calculate the rotation angles A and B of the waterjet. At present TruSurf can use two different methods to find the direction of the cutting vector. Method one sets the cutting vector orthogonal to both the surface normal and the surface tangent. This gives a very good approximation to the model shape. Method two sets the cutting vector as the line connecting the corresponding B-spline parameters on the top and bottom surfaces of each layer.

Figure 1 illustrates method one by showing a single layer of a model. Solid contours represent the path that would be cut by the cutting vector. These may or may not lie on the B-spline model surface. The dashed line is the contour calculated from the B-spline surface model. It is at a height midway between the top and bottom surfaces of each layer. This position was chosen to give the average slope over the layer. The direction of the cutting vector is found by computing the cross product of the surface normal and the tangent vector.

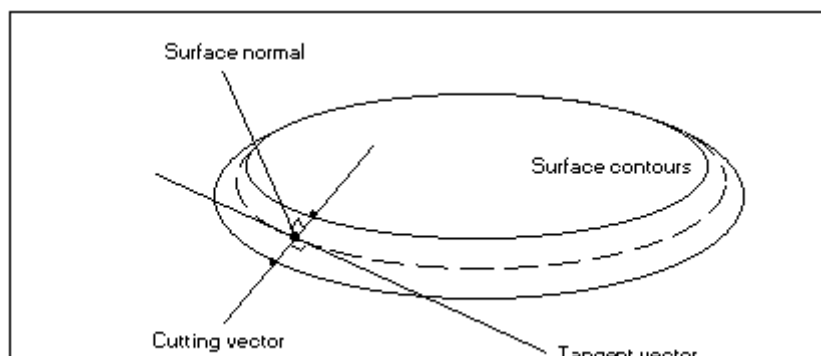


Figure 1. Illustration of method one, which uses a single interpolated contour plus the B-spline surface normal.

The tangent vector is the intersection of the tangent plane to the B-spline surface, and the model slicing plane. Its direction is calculated from the unit cross product of the normals of the two planes. The tangent plane to the surface is defined by the two vectors obtained from the partial derivatives of the surface P with respect to parameters s and t. The surface normal n is found from the cross product of these two vectors.

$$n = \frac{dP}{ds} \times \frac{dP}{dt} \quad (4)$$

From formal differentiation of equation (1) the derivatives of a rational B-spline surface are:

$$\frac{dP}{ds} = \frac{\overline{N}}{\overline{D}} \left( \frac{\overline{N}_s}{\overline{N}} - \frac{\overline{D}_s}{\overline{D}} \right), \text{ and } \frac{dP}{dt} = \frac{\overline{N}}{\overline{D}} \left( \frac{\overline{N}_t}{\overline{N}} - \frac{\overline{D}_t}{\overline{D}} \right) \quad (5)$$

where N and D are the numerator and denominator of equation (1), respectively, with derivatives,

$$\begin{aligned} \overline{N}_s &= \sum_{i=1}^{m-1} \sum_{j=1}^{m-1} W_i P_i b'_{i,j}(s) b_{i,j}(t) & , & & \overline{N}_t &= \sum_{i=1}^{m-1} \sum_{j=1}^{m-1} W_i P_i b_{i,j}(s) b'_{i,j}(t) & , \\ \overline{D}_s &= \sum_{i=1}^{m-1} \sum_{j=1}^{m-1} W_i b'_{i,j}(s) b_{i,j}(t) & , & & \overline{D}_t &= \sum_{i=1}^{m-1} \sum_{j=1}^{m-1} W_i b_{i,j}(s) b'_{i,j}(t) . \end{aligned} \quad (6)$$

Each prime denotes a derivative of the basis function with respect to its corresponding parametric variable. The derivatives of the basis functions are obtained by differentiation of the recursion formulas (2) and (3).

$$b'_{i,1}(s) = 0 \text{ for all } s, \quad (7)$$

and

$$b'_{i,k}(s) = \frac{b_{i,k-1}(s) + (s - x_i) b'_{i,k-1}(s)}{x_{i,k-1} - x_i} + \frac{(x_{i,k} - s) b'_{i,k-1}(s) - b_{i,k-1}(s)}{x_{i,k} - x_{i+1}} . \quad (8)$$

Method two is illustrated in figure 2. Here, the top and bottom contours of the layer are interpolated from the B-spline surface, and the direction of the cutting vector is determined by joining the points of equal s parameter on each contour. Because the cutting vector joins the top and bottom contours the outer surface of the fabricated prototype is continuous at the layer joins.

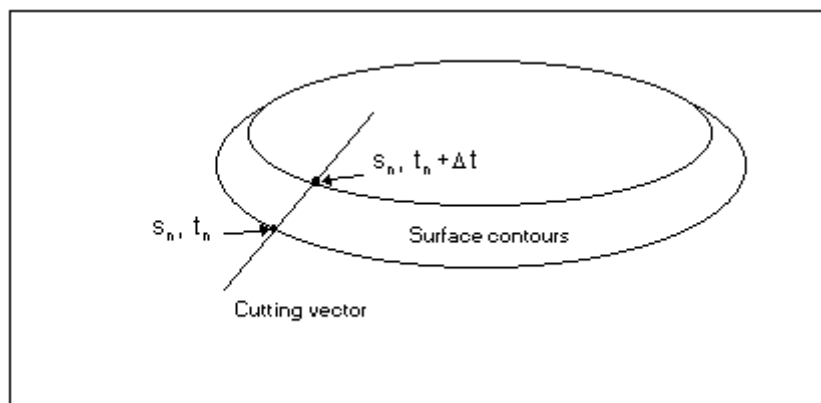


Figure 2. Illustration of method two, which uses two contours plus the B-spline parameters.

Figure 3 shows three front views of a test part. The left hand view is that of the B-spline surface. The middle and right views show how the part would look if made using methods one and two respectively. The difference between the two methods is most apparent in the top layer of the model. Method one uses the slope from the middle of the layer, resulting in a flat top surface, and overhang of the previous layer. Method two has no overhang, it joins the top of the previous layer to the top contour of the current layer. In this case the top contour of the original hemisphere has become a single point. The preferred method would be determined by the part, and its application.

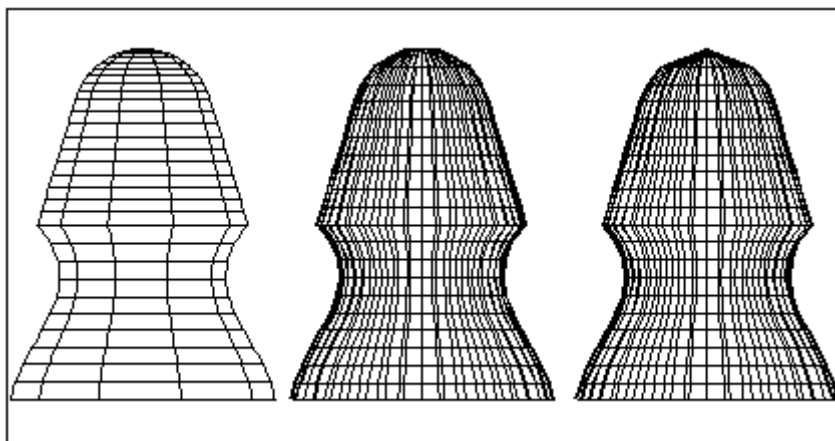


Figure 3. Front views of the first test part, illustrating the difference between methods one and two.

**SURFACE ERRORS**

Sloped surface layers are more accurate than stepped edged layers, but for a surface with double curvature there will still be some errors. Figure 4 illustrates the difference between the CAD model and the cut layers. In method one the errors are at the layer joins. For a convex surface extra material is left on the layer plane, while for a concave surface extra material is removed. The converse will hold for method two.

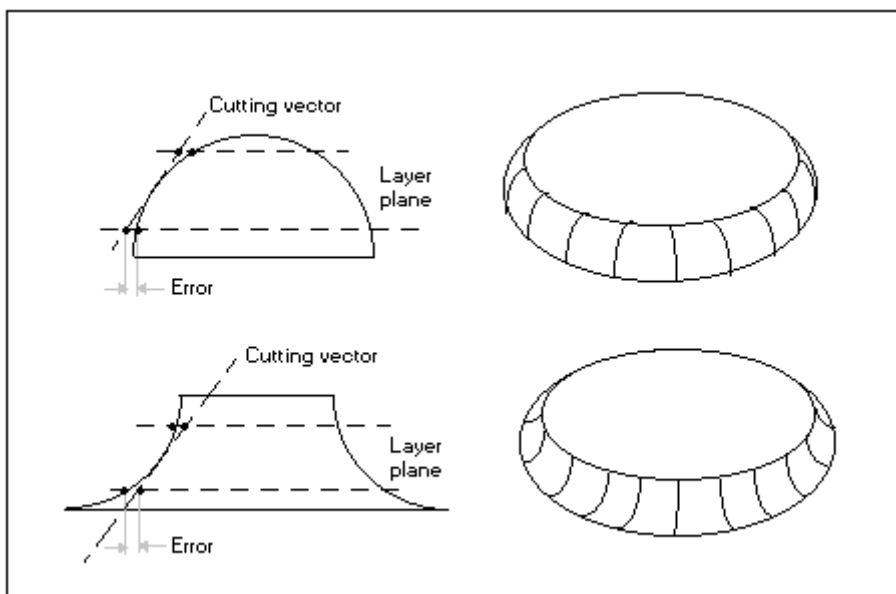


Figure 4. The error between the CAD model, and the cut layers.

For method one, the difference between where the cutting vector intersects the layer plane, and where the CAD model intersects the layer plane is dependent on three factors. Layer thickness, curvature of the surface, and the angle of the radius of curvature to the layer plane. The graph in figure 5 shows the error when the plane of maximum surface curvature is normal to the layer plane. The graph shows that the errors rapidly become larger as the radius of curvature becomes less than the layer thickness. However they are very good when the radius of curvature is equal to, or greater than, the layer thickness. Compared to stepped edge layers, these errors are quite small. For example, the

error on the tenth layer from the bottom, of the part in figure 3, for a stepped and a sloped prototype, will be 7.4 mm and 0.54 mm respectively.

In models of considerable size the radius of curvature is likely to be much greater than the layer thickness used. However there may be sections of a model that contain significantly more detail than the rest of the model. To allow for this it is possible to include an error checking algorithm in the layering software. Such an algorithm would allow the user to select an acceptable error tolerance, and then check each layer. If the error on any particular layer is unacceptable, the program could introduce sections with thinner layers to bring the error within the required tolerance.

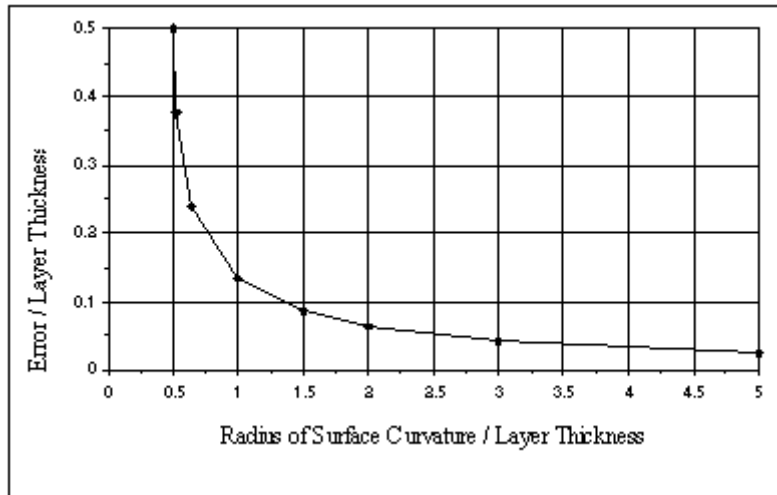


Figure 5. Graph representing relative errors as a function of radius of curvature normalised by layer thicknesses.

### EXAMPLE PARTS BUILT WITH TRUSURF

Two simple parts have been built using TruSurf. Polystyrene foam was chosen as the material due to its very low cost and ease of cutting. One 1500 x 2500 x 10 mm sheet cost AUD \$8. When measured it was found that the thickness of the sheet varied from about 10.1 mm to 10.2 mm. Thus it was decided to use the average layer thickness in TruSurf. The parts were created as surface models using the I-DEAS CAD package, and exported as IGES files. TruSurf sliced each part into nineteen 10.15 mm thick layers, and wrote the NC Code required to cut all thirty eight layers out of a single sheet of material. A 5-axis waterjet cutter was used to cut the material, and the layers were glued together by hand.

The two models produced with TruSurf are shown in figures 6 and 7. Figure 3 also shows a front view of the CAD surface model of the first part. The second part has an elliptical top and base. The top ellipse is smaller than the base, and is twisted through about 45°. The part also has a lean of about 15° and is similar in shape to a propeller blade.

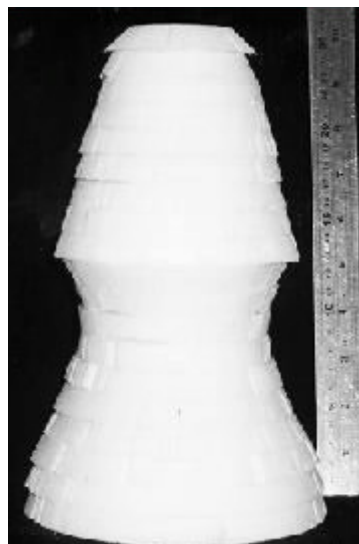


Figure 6. The first part built. It has 19 layers of 10mm thick foam. The rough surface was due to problems cutting the

foam.

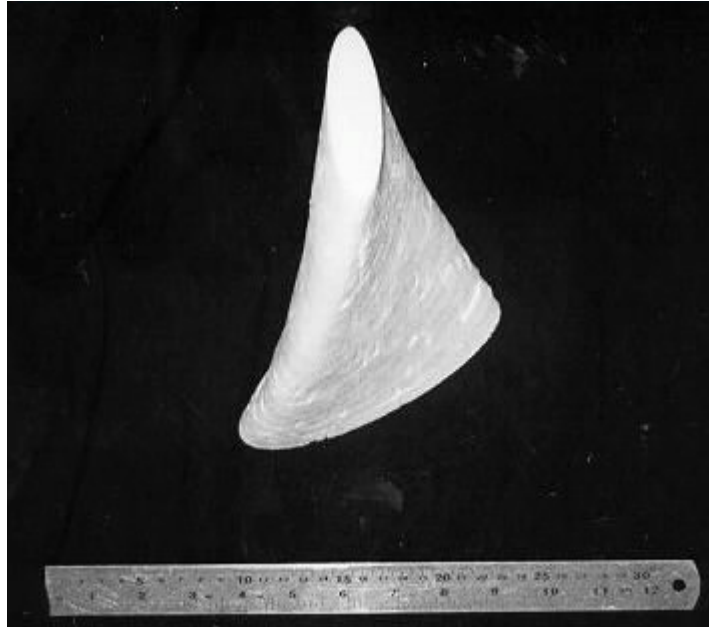


Figure 7. The second part built. More care was taken when cutting the foam, and a better surface was produced.

By using thicker layers and sloped surfaces an order of magnitude time saving was achieved. To build these two parts with a current rapid prototyping machine would take between ten and twenty hours. TruSurf took about two minutes to process each part, and it took less than an hour to cut and assemble the two parts. As mentioned above the main problem in making large prototypes is the build time. Thus, by producing the two parts in such a short time, TruSurf has shown the potential to work very well for making large prototypes.

When cutting the layers, some problems with using foam sheets on the waterjet cutter became obvious. The problems were mainly due to the light weight of the foam, and would not occur with heavier and more rigid materials. The effect of the problems can be seen in figure 6, which shows the first part that was produced. The large amount of discontinuity at the layer boundaries is due to the inconsistent height of the nozzle above the foam sheet. The gouges and spikes seen on the surface are due to the reflecting jet periodically lifting the foam. Figure 7 shows the second part to be cut. For this part more care was taken with the positioning of the foam on the grate, and the cut error was greatly reduced. Weights were also used to stop the foam sheet being lifted by the reflected water jet.

## PRACTICAL APPLICATIONS OF TRUSURF

The practical applications of TruSurf include those of other RP systems, such as producing parts for concept modeling, component testing, and further manufacturing operations. TruSurf was initially developed for prototyping of objects larger than one meter cubed. The major uses were seen in rapidly producing accurate large moulds for casting and forming processes. However the TruSurf process is also capable of producing smaller parts with more accuracy, or greater speed than stepped edge methods. TruSurf's heart is the slicing software, making it a very versatile process. The machine used to produce parts can be any four or five axis NC machine capable of cutting or forming layered material. A five-axis waterjet was used in this project because of its availability, and size. A laser or plasma cutter could easily be used, as could other existing 3-axis RP systems if more axes of control were added. Choice of material is only limited by the machine used. The waterjet can cut most materials, including foam, wood, plastics, glass, ceramics, and metal. Methods of bonding the layers still need to be addressed, particularly for metals and ceramics.

## CONCLUDING REMARKS

A new method for creating rapid prototypes with sloping layer surfaces has been presented. For the same layer thickness the surface accuracy can be an order of magnitude better than stepped edge models. When thicker layers are used the build time can be reduced by an order of magnitude over a stepped model with similar accuracy.

The next step in developing the system is to incorporate error checking, and the use of different layer thicknesses in the one model. The slicing software will automatically select thicker layers for sections with little surface curvature, and thin layers in areas of detail. This will optimise the building of parts for both speed and accuracy.

The use of principal directions to determine the cutting vector will be tried. At any point on a surface there are two directions in which the normal curvatures take extremum values (Hosaka, 1992). These directions are called the principal directions, and the curvatures are called the principal curvatures. If at least one of the principal curvatures is zero, the surface is developable, and a line of curvature on the surface is straight. Such surfaces could be cut by a straight medium without any error. In any case, one of the principal directions will give the direction of minimum surface curvature, and the use of a cutter orientated in this direction will produce the minimum possible error.

Another method to increase the accuracy of the fabricated prototype, is to use a curved cutting medium on the layer surfaces. Thus the layer surface would not only be sloped to match the surface slope, but curved to match the surface curvature. To achieve this a hot wire or thin laminate could be used to cut the layers. One method to curve the cutting medium is to move the controlling heads closer together, and cause it to bend. The amount of bending would depend on how far the heads are moved. However it may be difficult to control the direction in which the cutting medium bends. Alternatively the controlling heads could rotate the tips of the cutting medium to cause it to bend in the required direction. Another method is to use three points for the cutting medium. Figure 8 illustrates the points used.

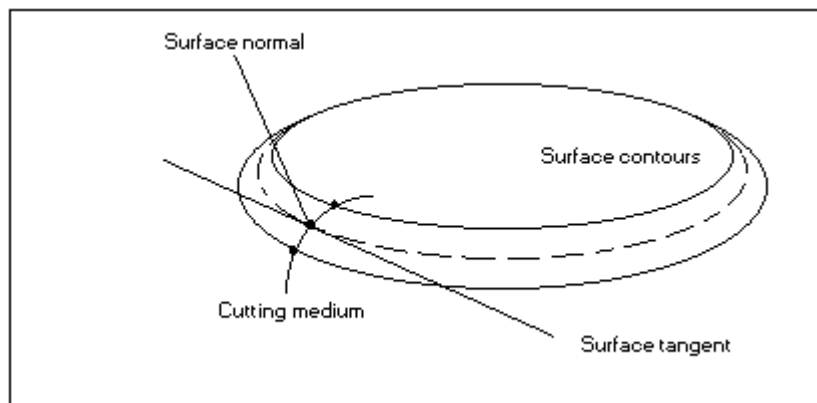


Figure 8. Using three points to cut a curved surface on layers.

As shown, three points on the surface are used for each layer, and each one is traced simultaneously to cut the layer. The main difficulty with curved cutting is that it requires the development of specialty apparatus, and more axes of control are needed. However the benefits in terms of surface accuracy are very significant.

## ACKNOWLEDGMENTS

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