Comparison of dynamic and static hot wire anemometer calibrations for velocity perturbation measurements

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Abstract The sensitivity of constant temperature hot wire anemometers to velocity perturbations in an air flow was measured by the traditional static calibration technique as well as by the recently revived dynamic method in which the wire is shaken in a steady flow. Contrary to previously published findings, the two methods gave calibration constants which agreed to within an experimental standard error of 2% in the velocity range of 3–32 m s^{-1}. A new signal processing technique was used for the dynamic calibration method which gave better accuracy in the presence of electronic noise and wind tunnel turbulence than the RMS method used to date. In order to eliminate effects due to end conduction and structural resonances, perturbation frequencies were limited to 15 Hz. Also, effectively 'long' and 'short' wires were tested as well as different wire attachment methods. Perturbation levels were limited to 9% of the mean velocity. In view of the significantly greater simplicity of the static calibration method, its continued use is recommended provided that empirical cooling laws appropriate to the range of velocity of interest are used.

1 Introduction

Two different methods of calibrating constant temperature hot wire anemometers for velocity perturbation measurements have been proposed and used by researchers. In the static technique the hot wire anemometer output E is expressed as an empirical function of the mean flow velocity U, as measured with a pitot tube and micromanometer. This function is differentiated, the slope $\partial E/\partial U$ being the static calibration coefficient or sensitivity of the hot wire to small velocity perturbations at a particular velocity. The dynamic technique involves shaking the wire at low frequencies in a uniform flow of velocity U thus subjecting the wire to the velocity perturbation u. The resultant anemometer output will consist of a mean voltage E and a simultaneous voltage perturbation e. A dynamic calibration coefficient can be defined as e/u or the ratio of the respective RMS values of the perturbations. If the perturbation level is sufficiently low and if during the perturbation cycle the wire is at all times in thermal equilibrium with its end supports, in the absence of other extraneous mechanical and electrical effects, equation (1) should apply.

$$\frac{\partial E}{\partial U} = \lim_{\substack{u\to 0\\ f\to 0}} \frac{e}{u} = \lim_{\substack{u\to 0\\ f\to 0}} \frac{\overline{e^2}^{1/2}}{\overline{u^2}^{1/2}} \tag{1}$$

where f is the perturbation frequency and a bar denotes time averaging. That is, the static and dynamic calibration coefficients should be equal. Such agreement has not been reported to date.

Perry and Morrison (1971) indicate that there can be a large difference in the calibration curves determined by the two methods and that a dynamic calibration is more accurate and consistent than a static one. Kirchhoff and Safarik (1974) also indicate that a different calibration curve is obtained by the two methods. However, their results indicate substantial agreement between the statically and dynamically measured calibration coefficients at low mean velocities, when using King's cooling law as the functional relationship between E and U in the static method.

The accuracy of the various functions relating E and U in the velocity range of 0–150 m s^{-1} was investigated by Bruun (1971) who found that the three-term function proposed by Siddall and Davies (1972) gave the best fit for the greatest part of this velocity range. Kinns (1973) proposed a new functional relationship which is relatively insensitive to errors in the experimental measurement of E and U. The author states that, in view of the accuracy obtained during tests with the method, the resultant calibration coefficient should equal that obtained by the dynamic method although no such tests were made. The carefully calibrated wires were, however, subjected to a cylinder wake test similar to that used by Perry and Morrison (1971). A similarity of profiles was achieved from which it may be concluded that the statically determined calibration coefficients were correctly measured by this method or all differed from the true value by the same factor. This latter doubt was not resolved. Furthermore, a comparison with the results of Bruun (1971) is not presented.

2 Theoretical aspects of the dynamic calibration method

Perry and Morrison (1971) and Kirchhoff and Safarik (1974) used the ratio of the respective RMS perturbation values of equation (1) to obtain the dynamic calibration coefficient but the former used the total $\overline{e^2}^{1/2}$ value whereas the latter by application of a lock-in amplifier used only that portion of $\overline{e^2}^{1/2}$ which occurred at the perturbing frequency. The results of the two methods lead only to partial agreement.

Expansion of e in a Taylor series as carried out by Kirchhoff and Safarik (1974) shows that at perturbation levels of 10% RMS relative to the mean, the contribution of the higher-order terms in the expansion is less than 1%. This is consistent with the empirical finding of Perry and Morrison (1971) that even at a 20% perturbation level, no measurable contribution occurs.

A drawback when using $\overline{e^2}^{1/2}$ by either method is that electronic and wind tunnel turbulence is often sufficiently large relative to the induced velocity signal that $\overline{e^2}^{1/2}$ for use in equation (1) must be found by subtracting the mean square noise from the mean square of the desired velocity signal with noise. It is assumed that the desired and undesired signals are uncorrelated. If these two mean squares are of similar order of magnitude, poor accuracy results. In addition, during the shaking of the probe, small wire or wire support vibrations can be introduced as well as unknown flow distortions. Even if these are uncorrelated with the desired signal, they will be included in the final $\overline{e^2}^{1/2}$ and lead to an error.

A method which is insensitive to extraneous uncorrelated signals uses the correlation $\overline{eu}$ as follows. Expanding e as a function of u in a Taylor series,

$$e = E'u + \frac{E''u^2}{2!} + \frac{E'''u^3}{3!} + \ldots \qquad (2)$$

where $E' = \partial E/\partial U$ and similarly for the higher-order derivatives E'', E''' ....

Multiplication of equation (2) by u, time averaging and dividing throughout by $\overline{u^2}$ yields

$$\frac{\overline{eu}}{\overline{u^2}} = E' + \frac{E''}{2} S\overline{u^2}^{1/2} + \frac{E'''}{6} F\overline{u^2} + \ldots \qquad (3)$$

where skewness $S = \overline{u^3}/\overline{u^2}^{3/2}$ and flatness $F = \overline{u^4}/\overline{u^2}^{2}$.

If the perturbation is symmetrical, $S=0$, and if it is also sinusoidal then $F=1{\cdot}5$. Substitution in equation (3) leads to

$$\frac{\overline{eu}}{\overline{u^2}} = E'\left(1 + \frac{E'''}{4E'}\overline{u^2} + \ldots\right). \qquad (4)$$

At a perturbation level as high as 10%, that is, $\overline{u^2}^{1/2}/U = 0{\cdot}1$, the second term in equation (4) is still less than 0·01 so that it may be neglected, leading to the following results:

$$E' = \frac{\overline{eu}}{\overline{u^2}} = \frac{\partial E}{\partial U} = \lim_{\substack{u\to 0\\ f\to 0}} \frac{e}{u}. \qquad (5)$$

e is the voltage available from the hot wire anemometer and a voltage proportional to u can be obtained quite easily with a displacement or velocity transducer attached to the hot wire probe stem. Experimentally, this method is no more difficult than the RMS method but has the advantage that all uncorrelated extraneous signal components contained in e are correlated out. This avoids the subtraction of quantities of similar order of magnitude as well as the need for accurate assessment of the total extraneous noise component. If the signal from the transducer giving u has a high signal to noise ratio, $\overline{u^2}$ does not require correction for noise.

So far, only the accuracy of signal processing and the requirement that the perturbation be small enough have been discussed. The requirement that $f \to 0$ also requires consideration.

Commonly used hot wire elements have a substantial heat loss to their supports which for a 1 mm × 5 μm tungsten element at low subsonic velocities in air is of the order of 50% of the total heat input. A frequently used assumption is that the ends of the wire are at a constant temperature equal to or, more generally, slightly above the stream temperature. Bremhorst *et al* (1976) have shown that this is not the case. Instead the temperature of the ends of the wire fluctuates with its own time constant which differs from that of the wire. Unless the perturbation frequency is below that corresponding to this time constant, thermal equilibrium will not exist between the wire and its supports. This will in turn change the ratio of heat lost from the wire by convection to that lost by conduction to the supports under dynamic conditions, from the ratio obtained in a static calibration. For the wires and supports used in the present work, perturbation frequencies had to be limited to 15 Hz to avoid these complications as well as excitation of structural resonances in the hot wire and its supports. Bremhorst *et al* (1976) have also shown that the effects on the wire response due to the end supports become insignificant for an effectively long wire, that is, one which has negligible heat losses to the supports. In the absence of other effects not accounted for, it is to be expected that at the perturbation frequencies used, $\partial E/\partial U = e/u = \overline{eu}/\overline{u^2}$; if not, then by reducing the perturbation frequency, this equality must be approached asymptotically as $f \to 0$.

3 Experimental apparatus and procedures

All tests were performed at the outlet of a 40 : 1 contraction from a settling chamber designed to produce a low turbulence flow free of swirl. A DISA 55M01 system with a 55M11 CTA bridge was used but some measurements were repeated with a locally built constant temperature anemometer. The two units gave identical results. Hot wire and probe details are shown in figure 1. The probe was designed for negligible flow distortion near the wire with the aid of hydrogen bubble flow visualization in a water tunnel. The hot wire was perturbed by an electrodynamic shaker with perturbation levels $\overline{u^2}^{1/2}/U$ always being below 9%.

Hot wire velocity perturbations were measured with a Hewlett–Packard type 6LV2 velocity transducer attached to the hot wire probe. The point of attachment was varied along the probe in case vibrational nodes existed along the probe stem. No effect on the results was noted. The velocity transducer was calibrated to an estimated standard error of better

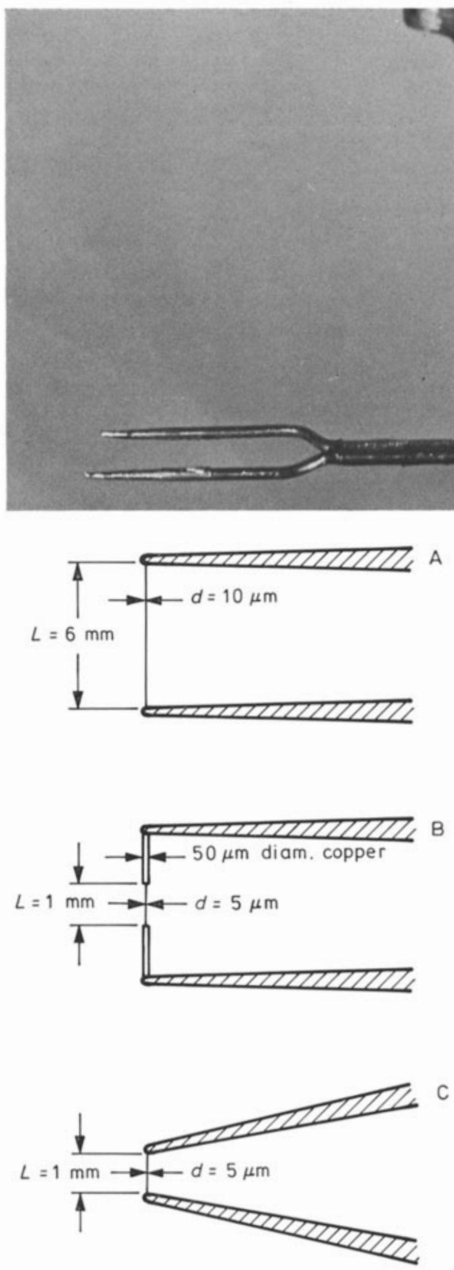

Figure 1 Hot wire and probe details. A, platinum–rhodium wire; B, tungsten wire with copper-plated ends; C, tungsten wire directly attached to supports

than 1% with a Hewlett–Packard type 7DCDT displacement transducer which in turn was calibrated with a vernier telescope. Flow velocities at the hot wire were measured with a pitot tube and Betz micromanometer. Signal processing was carried out on an EAI 231R analogue computer on-line.

4 Measured calibration coefficients and comparison with previous data

Figures 2 and 3 show the calibration coefficients obtained with the three types of wire of figure 1 using the dynamic and static

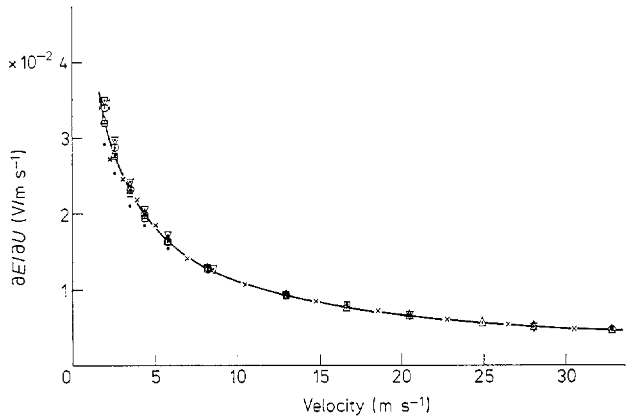

Figure 2 Measured calibration coefficients using tungsten wire, an effectively short wire. $d=5$ μm, $L=1$ mm, cold resistance = 3·84 Ω, overheat ratio = 0·8. —△— Siddall and Davies (1972), using both plated and unplated wires; ⊡ Bruun (1971); × Kinns (1973); • King's law (equation (6)); ⊙ dynamic calibration (wire with copper-plated ends); ▽ dynamic calibration (wire directly attached to supports)

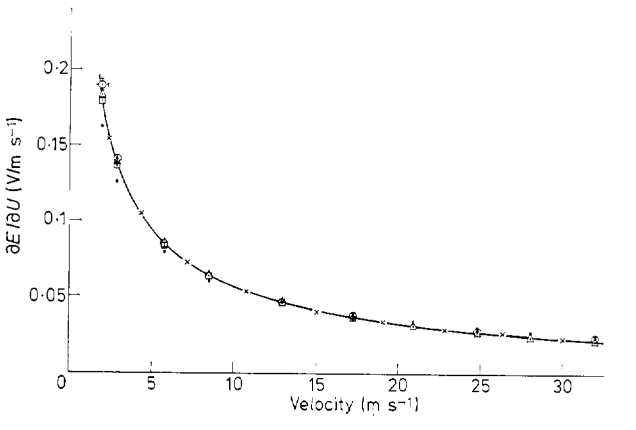

Figure 3 Measured calibration coefficients using platinum–rhodium wire, an effectively long wire. $d=10$ μm, $L=6$ mm, cold resistance = 19·28 Ω, overheat ratio = 0·295. —△— Siddall and Davies (1972); ⊡ Bruun (1971); × Kinns (1973); • King's law (equation (6)); ⊙ dynamic calibration

methods. For the latter, the following empirical functional relationships were used:

King's law,

$$E^2 = A + BU^{0\cdot5} \tag{6}$$

Bruun (1971) valid for the velocity range of 2–20 m s⁻¹

$$E^2 = A + BU^{0\cdot46} \tag{7}$$

Siddall and Davies (1972) valid for the velocity range of 0–160 m s⁻¹

$$E^2 = A + BU^{0\cdot5} + CU \tag{8}$$

Kinns (1973) valid for the velocity range of 2–60 m s⁻¹. At U_m,

$$\frac{\partial E}{\partial U} = \frac{\Delta E}{\Delta U}\left[1 + 0\cdot06\left(\frac{\Delta U}{U_m}\right)^2\right]^{-1} \tag{9}$$

where

$$U_m = \tfrac{1}{2}(U_1 + U_2),$$

$$\frac{\Delta U}{U_m} = \frac{2(U_2 - U_1)}{U_2 + U_1}$$

and

$$\frac{\Delta U}{U_m} < 0\cdot7.$$

Zero-velocity points were excluded when calculating the constants A, B and/or C in equations (6), (7) and (8).

In the velocity range 2–20 m s⁻¹, equations (7) and (8) agreed to within 2% of each other. Equations (8) and (9) agreed to within 1% in the range 3–35 m s⁻¹ but differed markedly at 2 m s⁻¹ for the tungsten wire. Indications are that the lower limit of validity of equation (9) should be 3 m s⁻¹ and will be assumed to be so in further discussions. King's law, equation (6), is seen to give only an approximate fit as reported by others.

The results of figures 2 and 3 show that the static and dynamic calibrations agree with each other to within the experimental standard error of 2% except at the low velocities where accuracy was limited by the micromanometer when measuring U with the pitot tube. This region of uncertainty is marked on figures 2 and 3. Since the calibration curve rises very steeply below 3 m s⁻¹, extremely precise measurements of U are required if the static method is to be used with confidence. For accurate turbulence measurements in this range, the present work indicates that calibration coefficients are more easily determined by the dynamic method but at higher mean velocities, the static method is far simpler to apply.

This excellent agreement between the static and dynamic calibration coefficients was unaffected by a change in perturbation frequency: up to 15 Hz; perturbation level: up to 9%; wire end configuration: copper-plated ends or direct attachment of wire to supports, and effectively long and short wires.

Some of the above dynamic calibration tests were also carried out by using the RMS method. Repeatability was much poorer than with the correlation technique but on the average did agree with the above results.

5 Discussion of results

As the present results are contrary to those of Perry and Morrison (1971) and Kirchhoff and Safarik (1974) further discussion appears justified. In the absence of radiation, thermoelectric, flow interference and stray mechanical effects due to poor probe design, and assuming that a correctly designed electronic unit is used, the only factors which could produce a fundamental disagreement between the static and dynamic methods of calibration appear to be (i) end effects due to the wire supports; (ii) fluid dynamic effects if the boundary layer growth around the wire is too slow; and (iii) redistribution of the temperature profile along the wire, that is, distributed effects. All three factors affect the transient response of the wire and would therefore be frequency dependent, thus leading to a frequency dependence of $\partial E/\partial U$. Judicious selection of the perturbation frequency for the dynamic tests would then be most important.

For commonly used hot wire sensors, separate tests using temperature perturbation have shown that dynamic prong effects, that is, effects due to the finite time constant of the bulk of the wire supports, are most noticeable in the 1–15 Hz range of perturbation frequencies which is precisely that used for the present results. In view of the absence of any noticeable variation in $\partial E/\partial U$ with perturbation frequency, the present results indicate that the dynamic prong effect, if present, is not sufficiently significant to affect measurements with the types of wire arrangement tested. This finding is also consistent with that of Comte-Bellot (1975) who showed that for a 5 μm tungsten wire in a 10 m s^{-1} air flow no change in wire sensitivity to heat transfer changes (produced by laser pulsations) exists from 3 Hz to near the wire's roll-off frequency; thereafter the situation depends on the electronic circuitry. The present results are further enhanced by the fact that effectively long and short wires were tested with the same equipment and procedure, giving equally good agreement between dynamic and static calibrations. It must be concluded, therefore, that the dynamic prong effect described by Smits and Perry (1975) is not of significance with the probes investigated.

Effect (ii) has been investigated by Bullock and Ledwich (1973) who solved the time-dependent Navier–Stokes and enthalpy equations numerically. These studies showed that for the normal range of conditions met in subsonic turbulence measurements, no effect due to the finite response time of the boundary layer will be obtained.

Effect (iii) has not been investigated in detail but would not be met until frequencies of the order of the wire's roll-off frequency are encountered.

6 Conclusions

The results presented show that hot wire anemometer calibration coefficients for velocity perturbation measurements in an air flow obtained by the static and dynamic calibration methods are identical to within an experimental standard error of 2% in the velocity range 3–32 m s^{-1}. This result is independent of effective wire length and type of end support but has been verified only for the dynamic range within which the wire is known to be in thermal equilibrium with its supports at all times. For accurate calibration coefficient determination by the static method below 3 m s^{-1}, more accurate measurement of the mean velocity than has been possible in the present work is required. In view of the simplicity of the static method, its continued use is recommended provided that functional relationships between E and U appropriate to the velocity range of interest are used. The newly developed correlation method of obtaining the dynamic correlation coefficient was found to give much better consistency of results than the RMS method.

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